Syllabus for Entrance Test for PhD in Mathematics

Research Methodology

Introduction of research methodology: meaning of research, objectives of research, types of research, significance of research, problems encountered by researchers in India; research problem: definition, necessity and techniques of defining research problem, formulation of research problem, objectives of research problem; research design: meaning, need and features of good research design, types of research designs, basic principles of experimental designs, design of experiments, synopsis design for research topic.

Mathematics

Analysis: Elementary set theory, finite, countable and uncountable sets, real number system as a complete ordered field, Archimedean property, supremum, infimum; sequences and series, convergence, limsup, liminf; Bolzano Weierstrass theorem, Heine Borel theorem; continuity, uniform continuity, differentiability, mean value theorem; sequences and series of functions, uniform convergence; Riemann sums and Riemann integral, improper integrals; monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral; functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems; metric spaces, compactness, connectedness; normed linear spaces.

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations; algebra of matrices, rank and determinant of matrices, linear equations; eigenvalues and eigenvectors, Cayley-Hamilton theorem; matrix representation of linear transformations; change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms; inner product spaces, orthonormal basis; quadratic forms, reduction and classification of quadratic forms.

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions; analytic functions, Cauchy-Riemann equations; contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, maximum modulus principle, Schwarz lemma, open mapping theorem; Taylor series, Laurent series, calculus of residues; monformal mappings, Mobius transformations.

Algebra: Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems; rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain; polynomial rings and irreducibility criteria; fields, finite fields, field extensions, Galois theory.

Topology: Basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

Ordinary Differential Equations (ODEs): Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs; general theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs): Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs; classification of second order PDEs, general solution of higher order PDEs with constant coefficients, method of separation of variables for Laplace, Heat and Wave equations.